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Annex No. 10 to the MU Directive on Habilitation Procedures and Professor Appointment Procedures

HABILITATION THESIS REVIEWER'S REPORT

Masaryk University	
Applicant	Mgr.Peter Šepitka, Ph.D.
Habilitation thesis	Riccati Matrix Differential Equations and Sturmian Theory for Linear Hamiltonian Systems
Reviewer	Prof. Roberta Fabbri, Ph.D.
Reviewer's home unit, institution	Dipartimento di Matematica e Informatica "Ulisse Dini", Università di Firenze, Italy

The habilitation thesis of Peter Šepitka is devoted to the study of Riccati matrix differential equations and Sturmian theory for linear Hamiltonain systems.
It consists of two chapters in which the author presents the results obtained in the period 2016-2021 and which are published in five papers, collected in an appendix, two as a single author and the other three co-authored by Roman Šimon Hilscher . There are also some additional results obtained by the author and co-authors more recently and some research directions and open problems presented in the final section of Chapter 2.
Therefore the thesis can be considered as an exended commentary to the attached papers because the author inserts the results into the literature and the current developments of the subject.

In Chapter 1 the theory of the Riccati matrix differential equation associated to a linear Hamiltonian system is considered. Many results in the literature are given under the so-called Legendre condition and the complete controllability (identical normality) assumption. If the linear Hamiltonian systems does not satisfy the complete controllability (identical normality) assumption the system can be in general uncontrollable or abnormal.
In the Chapter the theory and properties of genera of conjoined bases of a general non oscillatory and possibly abnormal linear Hamiltonian system for which the Legendre condition is not assumed are presented. The results obtained use the relationship between subspaces of solutions of linear control systems and orthogonal projectors verifying a differential equation of Riccati type. The content of the Chapter is based on the first two papers collected in the appendix.

Chapter 2 considers the related oscillation theory: the Sturmian separation and comparison theorems are considered using the properties of the symmetric solutions of the corresponding Riccati matrix differential equations. An important tool for the analysis is the comparative index which was introduced by Julia Elyseeva in 2007 for discrete symplectic systems and then generalized by her to the continuous case in 2016 and independently formulated by the author and Šimon Hilscher in 2017 and presented in the third of the five papers which constitute the content of the thesis. Such an index is used to approach the regular and singular, on unbounded intervals, Sturmian theory. In the Chapter the separation and comparison theorems for nonoscillatory linear Hamiltonian systems on unbounded intervals are given. A new concept of a multiplicity of a focal point at infinity for conjoined bases is examined and for the analysis an application of new transformation teorems for principal and antiprincipal solutions at infinity are considered. It is important to notice that the

developed theory does not require any controllability condition and that the results obtained in the Chapter are new even in the complete controllable case.

In my opinion the habilitation thesis of Peter Šepitka fulfils the hight standards of research in the field of qualitative theory of differential equations and the presented results provide a significant contribution to the theory of the oscillation for linear Hamiltonian systems.

From the analysis presented it seems that many of the results obtained can be used independently in other fields of mathematics such as in linear algebra , especially in matrix analysis and in mathematical analysis, for the study of linear control systems.

Moreover the really dense thesis is well and clearly written and organized. I highly appreciate the scientific activity of Dr. Peter Šepitka , whose scientific production is very impressive for quality and rating.

Reviewer's questions for the habilitation thesis defence (number of questions up to the reviewer)

1.At the end of Chapter 2 you mention some further research directions. Do you already focus your attention on a particular subject?

2. Have you already worked on the relationship between the comparative index and the Maslov index introduced by Arnold in the 60's and which is used for the geometric definition of the rotation number for random linear nonautonomous Hamiltonian system?

Conclusion

The habilitation thesis entitled "Riccati Matrix Differential Equations and Sturmian Theory for Linear Hamiltonian Systems" by Peter Šepitka **fulfils** requirements expected of a habilitation thesis in the field of Mathematics-Mathematical Analysis.

Date: November 3, 2021 Signature: _