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Annex No. 10 to the MU Directive on Habilitation Procedures and Professor Appointment Procedures

HABILITATION THESIS REVIEWER'S REPORT

Masaryk University

Applicant	Mgr. Peter Šepitka, Ph.D.
Habilitation thesis	Riccati Matrix Differential Equations and Sturmian Theory for Linear Hamiltonian Systems
Reviewer	Doc. RNDr. Petr Stehlík, Ph.D.
Reviewer's home unit, institution	University of West Bohemia, Pilsen, Czech Republic

Report

The author collects and sums up his results from the set of five recent papers which were published in 2017-2020 and extend the theory of linear Hamiltonian systems and Riccati matrix differential equations both in the standard setting of complete controllability as well as in the uncontrollable case. The contribution to the widely studied classical theory is delicate and the so-called comparative index of two conjoined bases is introduced and the interesting historical link to discrete symplectic difference equations is discussed. On the other hand, the theory and concepts of abnormal Hamiltonian systems and associated Riccati matrix differential equations are systematically developed and the thesis basically builds a compact theory. Both the case of abnormal systems satisfying the Legendre condition (nonoscillatory systems) as well as the case of abnormal systems when the Legendre condition is possibly violated (which includes oscillatory systems) are described in detail. Among other concepts and results, the set of conjoined bases with the same image is called genus. Particularly, an ordering among the genera can be found and they are consequently found to form a nice algebraic structure – a complete lattice.

I particularly appreciate the systematic approach which enabled the construction of a compact new theory which extend the existing and well-described relationships in the theory of linear Hamiltonian systems. It is thus fair to mention that Peter Šepitka have (co-)authored numerous (at least twelve) other papers which are directly related to the content of the thesis. The text is written in a dense, very clean and rigorous style. The complete lack of typos and inconsistencies is admirable. Consequently, both the width and depth of results situate the papers and the thesis to the active boundary of actual knowledge. This is nicely illustrated by the parallel and independent introduction and analysis of the (continuous) comparative index by Peter Šepitka and Roman Šimon Hilscher on the one hand and Julia Elyseeva on the other hand. I also appreciate a lot the effort to rephrase the results from a different point of view in the thesis. The structure of two chapters, the former related Riccati matrix differential equations and the latter containing results connected to the Sturmian theory for linear Hamiltonian systems, does not correspond to the timeline of the published papers but enables a reader an original insight to the new theory. I am sure that the author had to spend a nontrivial amount of energy. The quality of the outcome of this effort deserves praise as well.

On the other hand, I have several critical remarks which are all minor in their nature and mostly matter of style and mathematical culture. I am a bit sorry about the lack of a broader motivation. Despite the obvious effort to give historical notes, I believe that the presented complex theory and nontrivial result would greatly benefit, e.g., from the numerous physical systems which have the structure of linear Hamiltonian systems and a discussion about an occurrence of abnormal systems. In the same spirit, the illustrative examples which could considerably help a reader to understand complex results are missing in the main text, despite the fact that almost every attached publication has a special and very helpful section devoted to the illustrative examples. I am especially sorry for this given the high quality of several lectures by Peter Šepitka I have attended. The spontaneous answers to questions related to broader context were always part of nice discussions. Finally, I believe that a researcher of such qualities should have much wider spectrum of co-authors at this stage of his career. Apparently, this is a matter of opinion and it is also obvious that Peter Šepitka is a part of a research group in Masaryk University in Brno which represents the leading body in this area. Naturally, it makes the need for outside links less important. And the submitted thesis is a nice example of this fact.

Overall, I believe that this is a high-quality thesis. The unusual systematic approach and a construction of a new theory situates it way above the average standards of habilitation theses I have had the opportunity to read. I am very curious about the future direction of Peter Šepitka's research.

Reviewer's questions for the habilitation thesis defence (number of questions up to the reviewer)

1. If I understand the results correctly, the totally abnormal Hamiltonian systems with null matrices A, B, C would generate the Riccati matrix differential equation with the set of genera of conjoined bases forming a lattice corresponding to Euclidean subspaces of $\mathbb{R}^n$. Reversely, is it possible to generate a Hamiltonian system which would correspond to any of its sublattice? If so, can we describe their principal solutions and minimal principal solutions?

2. The results of the thesis have a nice connection to the discrete analogue of Hamiltonian systems, the relationship to linear control systems is also clear and the the thesis nicely explains how its novel results influence or are influenced by these two fields. The author, however, also mentions new results in linear algebra. I am particularly curious about those related to the Moore-Penrose pseudoinverse which replace naturally inverses in the case of abnormal Hamiltonian systems. Being very excited after the introduction in which they are mentioned, I haven't found this link in the thesis or the attached papers. I wonder if Peter Šepitka meant the comparative index by this statement or if there is some delicate detail which I have missed or overlooked.

3. Many physical systems have the structure of linear Hamiltonian systems. Is there a simple example of a physical system (or a system from a different area of science) which could be described with an abnormal linear Hamiltonian system?

4. Given the systematic approach of the author, I wonder what are his next steps. Are there natural directions or do the results basically complete all interesting questions in the theory of abnormal Hamiltonian systems?

Conclusion

The habilitation thesis entitled “Riccati Matrix Differential Equations and Sturmian Theory for Linear Hamiltonian Systems” by Peter Šepitka **fulfils** requirements expected of a habilitation thesis in the field of Mathematics – Mathematical Analysis.

Date: Signature:

26th October 2021 Petr Stehlík