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HABILITATION THESIS REVIEWER'S REPORT

Masaryk University	
Applicant	Mgr. Peter Šepitka, Ph.D.
Habilitation thesis	Riccati matrix differential equations and Sturmian theory for linear Hamiltonian systems
Reviewer	Prof. Dr Hermann Schulz-Baldes
Reviewer's home unit, institution	Friedrich-Alexander University Erlangen-Nürnberg Department of Mathematics

The habilitation thesis of Peter Sepitka is about finite-dimensional real linear Hamiltonian systems. These are a special kind of systems of ordinary differential equations in which there are two separate components - position and momentum - that appear in the differential equation only in a particular form so that the solutions conserve an associated symplectic form. Of particular importance are sets of solutions that form a conjoined basis, often also referred to as a Lagrangian solution as they span a maximally isotropic subspace at every point, at least in generic situations which are relevant for many applications such as matrix-valued Sturm-Liouville equations and block Jacobi operators. In these latter situations the systems depend on an extra external parameter (called the spectral parameter or the energy) with respect to which a certain intrinsic positivity holds. This is also referred to the Legendre condition. Another well-known element of great relevance for the analysis of linear Hamiltonian systems are associated Riccati matrix differential equations.

The thesis, however, has its focus on singular solutions which appear in uncontrollable (or abnormal) and nonoscillatory systems without Legendre condition. Then there is no spectral parameter available and hence only space (and no energy) oscillations can be studied. While such general singular systems did not find more standard applications yet, there seem to be some applications to the calculus of variations and optimal control problems with variable endpoints. This is not explained in detail in the thesis which is, in any case, clearly driven by intrinsic mathematical curiosity. The main novel contributions of the thesis concern

- the genus of conjoined bases

- the Sturm-Liouville oscillation theory (focal point analysis and comparison theorems)

both

- for uncontrollable and nonoscillatory Hamiltonian systems

- with and without a Legendre condition

- on bounded as well as on unbounded intervals.

The definition of the genus from a prior paper with Hilscher is extended to singular systems, and it is, moreover, shown that there is a natural structure of complete lattice on the set of all genus. Many of the results on oscillation theory involve the comparative index and its dual, as introduced by Elyseeva. Moreover, the multiplicity of a focal point at infinity is introduced and used. Also minimal principal and maximal antiprincipal solutions of the Hamiltonian systems naturally enter the arguments and statements. I judge that many of the results are substantial contributions to the field of singular linear Hamiltonian systems.

The whole thesis is of algebraic nature, geometric aspects are not touched upon and also not used to illustrate concepts and statements. Unfortunately this will likely limit the impact of the work on an international level - even the author seems to be aware of this short-coming. To my taste, the presentation of the introductory Chapters 1 and 2 is too technical and is actually difficult to access for readers that are not familiar with the terminology of the field (e. g. definitions of conjoined basis, controllability and nonoscillatory solutions can only be found in the later chapters using a search algorithm).

Albeit these critical comments, the habilitation thesis clearly documents the technical prowess of Peter Sepitka. He is without doubt able to carry out independent mathematical research at a high technical level. He is aware of most of the abundant literature. Without hesitation, I can suggest the Faculty of Science of the Masaryk University to accept this as the written part of the habilitation.

Reviewer's questions for the habilitation thesis defence (number of questions up to the reviewer)

1. The quadratic form J appearing in the equation (H) on page 1 of the thesis plays a central role throughout. Which of the many subspaces appearing later on are Lagrangian (maximally isotropic) with respect to this form, and which are merely isotropic with respect to this form?

2. Is there a geometric interpretation of the comparative index?

Conclusion

The habilitation thesis entitled "Riccati matrix differential equations and Sturmian theory for linear Hamiltonian systems " by Dr. Peter Sepitka fulfils requirements expected of a habilitation thesis in the field of Mathematics – Mathematical Analysis.

Date: 18/10/2021 Signature: