Habilitation Thesis Reviewer's Report

Masaryk University	
Faculty	Faculty of Science
Procedure field	Mathematics - Geometry
Applicant	Mgr. Lenka Zalabová, Ph.D.
Applicant's home unit, institution	University of South Bohemia in České Budějovice
Habilitation thesis	Filtered Manifolds with Distinguished Transformations and Transformation Groups
Reviewer	Joseph A. Wolf, Ph.D.
	Professor in the Graduate School
Reviewer's home unit, institution	Department of Mathematics University of California at Berkeley

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To the Habilitation committee:

This is my report on Dr. Lenka Zalabová's habilitation thesis.

Dr. Zalabová's has done important research on Cartan geometry, specifically symmetries of filtered manifolds with emphasis on parabolic geometry. While this sounds rather technical, the fact is that it is an area of wide scope and increasing development. Filtrations came into geometry in the 1950's in Kuranishi's theory of prolangations, which applied bundle–theoretic methods to PDE on manifolds. The special case of parabolic geometry has important applications in algebraic and differential geometry, in group representation theory and harmonic analysis, and in various cohomology and category theories, and it clarifies aspects of classical conformal geometry. My work has mostly been in the interplay of geometry and group theory, where I used real group orbits on complex flag manifolds to study CR geometry and construct irreducible representations of semisimple Lie groups, so I can write principally from that viewpoint.

The theory of parabolic subgroups and parabolic subalgebras began with the 1954 thesis of Jacques Tits. He showed that compact homogeneous Kaehler manifolds, complex Riemannian symmetric spaces and homogeneous conformal manifolds all were special cases of homogenous spaces G/P where G is a complex semisimple Lie group and P is a parabolic subgroup. Using the root space structure of the Lie algebra $\mathfrak{g}$ of G as in a theorem of Kostant in my 1966 book on spaces of constant curvature, it is immediate that parabolic subalgebras correspond to filtrations. In the 1990's a number of special cases were developed by various Japanese mathematicians, but these were not well known in Europe and America. The 1997 book of Sharpe made Cartan geometry available to a number of working mathematicians, and the field developed with the school of Čap, Slovák, Eastwood and Souček. Finally, the 2009 book of Čap and Slovák filled out the basic theory and made parabolic geometry available to the general mathematical community. Dr. Lenka Zalabová started research while that book was being completed, and since then she has been an important developer in the field. Many important facts on symmetries of parabolic geometries are her research results. Most of them required new ideas and technical insight, and they show that she has the mathematical strength to continue development of her work on symmetries of parabolic geometries and related topics.

In order to understand geometry and analysis on a manifold with structure it is important to understand its symmetry group. That group can vary with the type of structure under consideration. Many different structures can be possible, and the choice of structure depends which of the many applications are being studied. For example on spheres S^n the various structures can lead to orthogonal groups, unitary groups, symplectic groups, and a certain exceptional Lie group. For parabolic geometries in general this is an important, relatively recent, area of study. It holds much promise for the future and I am sure that Dr. Zalabová will be prominent in that future.

Finally, let me point out that in addition to her single–author papers, Dr. Zalabová has had fruitful research collaborations with Jan Gregorovič, Boris Kruglikov, Henrik Winther, Jaroslav Hrdina, Vojtech Zadnik and Aleš Návrat. At the Simons Semester in Warsaw a year ago she collaborated informally with many participants, particularly Pavel Nurovsky and Michael Eastwood. Thus I have every reason to think that she will continue to be an important researcher and a valued member of the mathematical community.

Joseph A. Wolf
Professor in the Graduate School

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Reviewer's questions for the habilitation thesis defence (number of questions up to the reviewer)

In the context of graded Lie algebras and parabolic geometries, what is the significance of the representation of g_0 on the other g_i ?

Conclusion

The habilitation thesis entitled "Filtered Manifolds with Distinguished Transformations and Transformation Groups" by Mgr. Lenka Zalabová, Ph.D. *fulfils* requirements expected of a habilitation thesis in the field of Mathematics - Geometry].